Solve $0=3x^3-13x^2+8x+12$ given that $(3x+2)$ is a factor.

$3x+2=0$

$3x=-2$

$\therefore x=-2/3$

$$\begin{array}{r} 1x^2-5x+6 \\ 3x+2\overline{)3x^3-13x^2+8x+12} \\ \underline{3x^3+2x^2} \\ -15x^2+8x \\ \underline{-15x^2-10x} \\ +18x+12 \\ \underline{18x+12} \\ 0 \end{array}$$

$$\begin{array}{r|rrrr} -2/3 & 3 & -13 & 8 & 12 \\ & & -2 & 10 & -12 \\ \hline & 3 & -15 & 18 & 0 \end{array}$$

$(x+\frac{2}{3})(3x^2-15x+18)$

$(x+\frac{2}{3})(3)(x^2-5x+6)$

$0=(3x+2)(x^2-5x+6)$

$3x+2=0$ $(x-2)(x-3)$

$x=-\frac{2}{3}$ $x-2=0$ $x-3=0$

$x=2$ $x=3$

$\{-2/3, 2, 3\}$

Algebra III
Rational Zero Theorem

Name: ________________
1/8/18

Warm-Up:

Factor completely $f(x)=x^3-x^2-x+1 = (x + \#)(x + \#)(x + \#)$

Only options for zeros: ± 1

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -1 & 1 \\ & & -1 & 2 & -1 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$f(x)=(x+1)(x^2-2x+1)$

$=(x+1)(x-1)(x-1)$

Rational Zero Theorem

Let $f(x) = qx^n + a_1x^{n-1} + ...+ a_nx + p$

The list of possible rational zeros for $f(x)$ can be found by listing $\frac{factors(p)}{factors(q)}$.

Note: Not all zeros are rational!

Ex #1: List all possible rational zeros for $f(x) = 1x^3 + 7x^2 + 4x - 12$

(x)(x)(x)

poss. zero's = { factors of 12

$= \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

You try: List all possible rational zeros for $f(x) = x^3 + 2x^2 - 8x - 16$

Poss zero: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

Ex #2: List all possible rational zeros for $f(x) = 4x^3 - 7x + 8$

$$\text{Poss Zero} = \frac{\text{Factors } 8}{\text{factors } 4} = \pm\left\{\frac{1, 2, 4, 8}{1, 2, 4}\right\}$$

$$\left\{ \begin{array}{l} \pm 1, \pm 2, \pm 4, \pm 8 \\ \pm \frac{1}{2}, \cancel{\pm \frac{2}{2}}, \\ \pm \frac{1}{4} \end{array} \right.$$

Ex #3: Find all real zeros for $f(x) = x^3 + 2x^2 - 5x - 6$

poss Zero: $\pm 1, \pm 2, \pm 3, \pm 6$

$$\cancel{\begin{array}{r|rrrr} 3 & 1 & 2 & -5 & -6 \\ & & 3 & 15 & \\ \hline & 1 & 5 & & 0 \end{array}}$$

$x = \{-3, -1, 2\}$

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -5 & -6 \\ & & -3 & 3 & 6 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$(x+3)(x^2 - x - 2)$

$(x+3)(x-2)(x+1)$

$x + 3 = 0$ $\quad x - 2 = 0$ $\quad x + 1 = 0$

$x = -3$ $\quad x = 2$ $\quad x = -1$

You try: Find all real zeros for $f(x) = 4x^3 - 7x + 3$

Zeros $= \pm \dfrac{1, 3}{1, 2, 4}$

$\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{3}{2}, \pm \frac{3}{4}$

$$\begin{array}{r|rrrr} -1 & 4 & 0 & -7 & 3 \\ & & -4 & 4 & 3 \\ \hline & 4 & -4 & -3 & 9 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 4 & 0 & -7 & 3 \\ & & -12 & & \\ \hline & 4 & -12 & & \end{array}$$

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -7 & 3 \\ & & 4 & 4 & -3 \\ \hline & 4 & 4 & -3 & 0 \end{array}$$

$(x-1)(4x^2+4x-3)$

$(x-1)(2x+3)(2x-1)$

$x-1=0 \quad 2x+3=0 \quad 2x-1=0$

$x=1 \quad x=-\frac{3}{2} \quad x=\frac{1}{2}$

$x: \left\{-\frac{3}{2}, +1, \frac{1}{2}\right\}$

Ex #5: Find all real zeros for $f(x) = 4x^3 + 8x^2 - 2x - 4$

Poss: $\pm \dfrac{1, 2, 4}{1 \quad 2 \quad 4}$

$\pm 1, \pm 2, \cancel{\pm 4}, \pm \frac{1}{2}, \pm \frac{1}{4}$

$$\begin{array}{r|rrrr} -2 & 4 & 8 & -2 & -4 \\ & & -8 & 0 & 4 \\ \hline & 4 & 0 & -2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -4 & 4 & 8 & -2 & 4 \\ & & -16 & & \\ \hline & 4 & -8 & & \end{array}$$

$(x+2)(4x^2-2)$

$a=4 \quad b=0 \quad c=-2$

$x+2=0$

$x=-2$

$\frac{0}{8} \pm \frac{\sqrt{0^2-4(4)(-2)}}{8}$

$0 \pm \frac{\sqrt{32}}{8}$

$0 \pm \frac{4\sqrt{2}}{8}$

$\pm \frac{\sqrt{2}}{2}$

$x = \left\{-2, \pm \frac{\sqrt{2}}{2}\right\}$